MAXIMUM LIKELIHOOD ESTIMATION AND OPTIMAL DESIGN IN CONSTANT ACCELERATED LIFE TESTS FOR THE GENERALIZED BURR DISTRIBUTION WITH TYPE-I CENSORING*

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The present paper deals with the case of Constant-Stress Fully Accelerated Life Testing (CSFALT) when three stress levels are involved under mixture distributions with type-I censoring where a pre-specified censoring time is involved. The lifetimes of test are assumed to follow the Generalized Burr lifetime distribution. Maximum Likelihood (ML) method is used to estimate the parameters of CSFALT model. In addition, confidence intervals for the model parameters are constructed. Optimum CSFALT plans, that determine the best choice of the proportion of test units allocated to each stress, are developed. Such optimum test plans minimize the Generalized Asymptotic Variance (GAV) of the ML estimators of the model parameters. For illustration, numerical examples are given.

1-Introduction:

In many problems of life testing , the lifetime of a product or material with high reliability requires an unacceptably long period of time to acquire the test data at the specified use condition. So, life testing at normal conditions makes the test impracticable. For this reason, Accelerated Life Test (ALT) is the suitable and reasonable procedure to be applied. ALT is used to get quick information on the reliability of product components and materials.

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In (ALT) the units are tested under conditions that are more severe than the normal ones to induce failures of very high reliability systems in a short time. The main reason for accelerated tests is to estimate quickly information about a device under accelerated conditions and the information obtained from these tests is extrapolated, through a physically reasonable statistical model, to obtain information at normal conditions. This model is usually derived from an analysis of the physical mechanisms of failure of the device under test. It is assumed that changing the stress from one level to another affects the value of the parameters only and not the functional form of the lifetime distribution, this is a major assumption of ALT.

Several models are available in the literature concerning the relationship between certain parameters of the life time distribution and the stress levels at which the experiment is conducted. The power rule model is the most widely used model as an acceleration function.

The current approach to the problem of ALT involves building a model that consists of:

- A life distribution $f(t,\theta)$ that represents the time to failure of an item at risk where $\underline{\theta}$ is a victor of unknown parameters.
- A functional relationship $,\theta = g(\underline{s},\underline{\alpha})$,where $\underline{\alpha}$ is a vector of unknowns and $\underline{s}$ denotes the vector of stresses. It is assumed that changing $\underline{s}$ affects the value of θ only and not the functional form of $f(t,\theta)$.

There are different models showing how the stress $\underline{s}$ is affecting the failure distribution. Among these models, the most famous ones are the inverse power law, the Arrhenius, the Erying relationships and the log linear relationship.

The Inverse Power Law:
This model is mostly used for flash lamps and simple fatigue due to mechanical loading. This relation is given by :

$\theta = v / s^{p}$,

where θ is a parameter of life distribution, s is the applied stress, ν is the constant of proportionality and p is the power of the applied stress, where ν and p are the parameters to be estimated.

Accelerated life testing results are used in the reliability-design process to assess or demonstrate component and subsystem reliability and detect failure models. The causes of failure of a product are accelerated by increasing the applied stress above its usual value, There are two different methods of accelerating a reliability test: Increasing the use-rate of the product or increasing the aging-rate of the product (overstress testing).

As Nelson (1990)[1] indicates, the stress can be applied in various ways, commonly used methods are constant stress, step stress and progressive stress level. These kinds of stresses would induce early failures of the tested units.

In a constant stress accelerated test, each unit in the experiment is run under a prespecified constant stress level. A sample size of n units is divided into k groups, n_j, $j=1,2,\ldots\ldots,k$, where n_j units are all run under a constant stress c_j and $n=\sum_{j=1}^{k} n_j$. It is assumed that $c_1 < c_2 < \ldots\ldots < c_k$.

The acceleration model which is a relationship between stress and one or more parameters of the lifetime distribution must be chosen.

Life testing is the case where items taken from a population are put to test and their times to failure are noted. The case which implies observing the lifetime of all the items is called uncensored data, but such situation rarely happens in reliability testing. Then for the limited time or budget, the test must be terminated before the failure of all items. In life testing, the experiment is terminated by two common types of data censoring .The observations of the censored sample occur in an ordered manner. The most common life test experiments are: Testing is terminated after a prespecified number of failure r have occurred from all items of test n, where $r<n$, in this case the number of failures r is a fixed constant and time t is the random variable (type II censoring), or testing is terminated when all the items have failed or at a predetermined time t, whichever is sooner, in this case the number of failure r is the random variable and the time t is a fixed constant (type I censoring).

One method of constructing a new distribution is to use the known parametric form of a distribution and allow one (or more) of the parameters to vary according to a special probability law. The new distribution is called a Mixture of distribution. This theory has useful applications in industrial reliability and medical survivorship analysis.

If $f(t|\theta)$ is a probability density function depending on a m dimensional parameter vector $\underline{\theta}$ and if $G(\underline{\theta})$ is called a m-dimensional cumulative distribution function, then :

$f(t) = \int_{\underline{\theta}} f(t|\underline{\theta})g(\underline{\theta})$ is called a mixture density, and g($\underline{\theta}$) called the mixing distribution [2].

Dubey (1968)[3] obtained a (generalized Burr) distribution by mixing the Weibull distribution in the form

$$f(t|\phi,\theta)=\phi\theta\, t^{\phi-1} e^{-\theta t^{\phi}}, \qquad t>0,\ \phi,\theta>0,$$

over the Gamma distribution in the form :

$$g(\theta|\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}, \qquad \theta>0,\alpha,\beta>0$$

The resulting probability density function (pdf) has the following form:

$$f(t|\alpha,\beta,\phi) \quad = \quad \frac{\alpha\,\beta^{\alpha}\,\phi\,t^{\phi-1}}{(\beta+t^{\phi})^{\alpha+1}}, \qquad t>0,\ \phi,\alpha,\beta>0,$$

which is a generalized Burr distribution with three parameter (α,β,ϕ).

The distribution function is:

$$F(t|\alpha,\beta,\phi) \quad = 1-\left(1+\frac{t^{\phi}}{\beta}\right)^{-\alpha}, \qquad t>0\ .$$

The reliability function has the following form:

$$R(t|\alpha,\beta,\phi) = \left(1+\frac{t^{\phi}}{\beta}\right)^{-\alpha}, \qquad t>0\ .$$

and the hazard rate function is

$$h(t) = \frac{\alpha\phi\,t^{\phi-1}}{\beta+t^{\phi}}, \qquad t>0\ .$$

It was stated by Lewis (1981)[4] that many standard theoretical distributions, such as exponential, Weibull, logistic, normal , and Pareto are special cases or limiting cases of the Burr system of distributions. This can be investigated as follows:

1- $f(t|\alpha,\beta,\phi) = \dfrac{\alpha\phi\beta^{\alpha}t^{\phi-1}}{\left(\beta+t^{\phi}\right)^{\alpha+1}}$. Generalized Burr distribution (α,β,ϕ).

If $\phi=1$

Then $f(t|\alpha,\beta) = \dfrac{\alpha\beta^{\alpha}}{(\beta+t)^{\alpha+1}}$, which is Pareto distribution (α,β).

2- $$f(t|\alpha,\beta,\phi) = \frac{\phi\alpha t^{\phi-1}}{\beta+t^{\phi}}\left[\frac{\beta}{\beta+t^{\phi}}\right]^{\alpha}$$

$$= \frac{\alpha\phi\, t^{\phi-1}}{\beta+t^{\phi}}\left[1+\frac{t^{\phi}}{\beta}\right]^{-\alpha}$$

let $\theta = \frac{\alpha}{\beta}$,

then

$$f(t|\alpha,\beta,\phi) = \phi\, t^{\phi-1}\left[\frac{\alpha}{\beta+t^{\phi}}\right]\left[1+\frac{\theta\, t^{\phi}}{\alpha}\right]^{-\alpha}$$

$$= \phi\, t^{\phi-1}\left[\frac{\alpha}{\beta+t^{\phi}}\right]\left[1+\frac{\theta\, t^{\phi}}{\alpha}\right]^{-\alpha\frac{\theta t^{\phi}}{\theta t^{\phi}}}$$

$$= \phi\, t^{\phi-1}\left[\frac{1}{\frac{1}{\theta}+\frac{t^{\phi}}{\alpha}}\right]\left[\left(1+\frac{\theta\, t^{\phi}}{\alpha}\right)^{\frac{\alpha}{\theta t^{\phi}}}\right]^{-\theta t^{\phi}}$$

if $\alpha,\beta \to \infty$, it is known that $e = \lim_{t\to\infty}\left(1+\frac{1}{t}\right)^{t}$.

Then $\lim_{\alpha\to\infty} f(t|\alpha,\beta,\phi) = \phi\,\theta\; t^{\phi-1}\, e^{-\theta t^{\phi}}$,which is Weibull distribution (ϕ,θ).

3- let $\phi = 1$

Then $f(t|\theta) = \theta\, e^{-\theta t}$, Exponential distribution (θ).

The hazard function is considered in the choice of the distribution for survival or reliability data. The shape of the hazard function reflects type of risk to which the population under study is exposed as a function of time.

As Abd EL Wahab (2001)[5] indicated, the Burr type XII distribution {Burr (b,λ)} where $h(t) = \dfrac{b\lambda t^{b-1}}{1+t^{b}}$,for finite λ and if 0 $< b \leq 1$ the hazard function decreases with increasing t and ultimately approaches zero. For $b > 1$ the hazard function, $h(t)$, has an inverse u-shape. The hazard rate initially increases , attains a maximum at $t^{*} = (b-1)^{1/b}$ and then decreases to zero as $t \to \infty$.

The outline of the paper is as follows. Beside this introductory section, the paper incloude five section. Section 2 deals with the derivation of the maximum likelihood estimators of the Generalized Burr distribution. The confidence limits of the parameters are presented in section 3. Section 4 studies the optimum constant-stress test plans of the fully accelerated life testing (FALT). For illustration, simulation studies are given in section 5.

2- Maximum Likelihood Method:

Maximum Likelihood method has been widely considered as one of the most reliable ways to estimate the parameters of distribution. The ML method is commonly used for most kinds of censored data and the analysis of accelerated life tests.

The methodology is to perform k independent life tests at k values of stresses c. After observing the failure times at each stress level; the likelihood of the model parameters is formulated in terms of the data from all the k trials.

Once the MLE of the model parameters are obtained, the value of the scale parameter of generalized Burr distribution under usual condition is observed. The reliability function is estimated at a normal stress level c_u.

The ML methods are mostly used for most theoretical models and different types of censored data. MLE have suitable statistical characteristics. Although the exact sampling distribution of MLE are sometimes not determined, it is known that under appropriate regularity conditions, MLE are consistent and asymptotically normally distributed. Also, MLE have the invariance property. This property is helpful for estimating model's parameters and measurements. As an example of such measurements is the reliability function at a certain mission time.

Unfortunately, the MLE do not always exist in closed form and therefore, numerical techniques are used to compute estimates. The Newton –Raphson procedure is regarded as one of the most efficient numerical techniques so it is widely used.

There is a large amount of literature applying ML on estimation under Accelerated Life Testing for its massive applications in different fields. In the case of constant stress, Singpurwalla (1971)[6] has obtained a ML estimator of the mean life time of exponential distribution considering the inverse power law model.

A numerical scheme for solving ML equations was given by McCool (1980)[7] assuming that the Weibull scale parameter varies inversely with a stress variable.

Abdel-Ghaly (1981)[8] has generalized the work of Singpurwalla (1971)[9] for the case of the Weibull distribution with known shape parameter.

The MLE of aWeibull regression model under type-I censoring were derived by Bugaighis (1990)[10]. Moreover, bias and mean square error of the parameters are reported.

Using the generalized Burr distribution, the problems of both maximum likelihood estimation and optimal design for constant-stress FALT were studied by Abdel-Ghaly, *et al.* (2007) [11] using type II censoring.

In accelerated testing, experiments are usually terminated before all units fails. Censored data reduce test time and expense. Failure-censored data (type-II) are usually used in the theoretical literature but Time-censored data (type-I) are common in practice.

2.1 Maximum Likelihood Estimation With Type-I Censoring:

Let the life time experiment is assumed under k levels of high stresses $c_j, j=1,2,\dots\dots\dots,k$ and assume that c_u is the normal use condition such that $c_u < c_1 < c_2 < \dots\dots < c_k$, and there are n_j units are put on test at each c_j, $j=1,2\dots,k$. When a type-I censoring is applied at each stress level, the lifetime at stress c_j, t_{ij}, $i=1,2,\dots\dots,n_j$, $j=1,2,\dots\dots,k$, are assumed to be realizations from generalized Burr distribution with the density function.

$$f(t_{ij};\alpha,\beta,\phi_j)=\frac{\alpha\beta^{\alpha}\phi_j\, t_{ij}^{\phi_j-1}}{\left(\beta+t_{ij}^{\phi_j}\right)^{\alpha+1}},$$

$$t_{ij}>0,\beta,\alpha,\phi>0\ ,j=1,2,\dots k, and\, i=1,\dots\dots,r_j \qquad (2.1)$$

It is assumed that the stress c_j affects only on the scale parameter of the generalized Burr distribution ϕ_j through a certain acceleration model. The accelerated model is the model relating one parameter to the stress levels applied to the items being tested. Selection of this model is the most serious difficulty. This model should be physically reasonable for the particular item or product being tested and the kind of stress being applied to accelerated failures.

The inverse power law model suggested by Singpurwalla (1971)[12] will be considered. This model is widely used for electrical insulation in voltage-endurance tests, flash lamps and simple metal fatigue due to mechanical loading. It assumes the following relation:

$$\phi_j = v\, s_j^p \qquad , j=1,2,\ldots\ldots,k. \tag{2.2}$$

Where v is the constant of proportionality and p is the power of applied stress are the parameters of this model such that .

$$s_j=\frac{c^*}{c_j} \quad , c^* = \prod_{j=1}^{k} c_j^{b_j} \quad , b_j=\frac{n_j}{\sum_{j=1}^{k} n_j} \quad ,v>0 \ ,p>0 \ .$$

Applying type-I censoring at each stress level, the experiment once all the items fail or when a fixed censoring time L_j is reached.

Then the corresponding likelihood function is expressed as follows:

$$L(v,p,\alpha,\beta|\underline{t})=\prod_{j=1}^{k}\prod_{i=1}^{n_j}\left\{\left[\frac{\alpha\beta^{\alpha}\phi_j\, t_{ij}^{\phi_j-1}}{\left(\beta+t_{ij}^{\phi_j}\right)^{\alpha+1}}\right]^{\delta_{ij}}\left[\left(1+\frac{L_j^{\phi_j}}{\beta}\right)^{-\alpha}\right]^{1-\delta_{ij}}\right\}$$

$$=\prod_{j=1}^{k}\prod_{i=1}^{n_j}\left\{\left[\frac{\alpha\beta^{\alpha}vs_j^p\, t_{ij}^{vs_j^p-1}}{\left(\beta+t_{ij}^{vs_j^p}\right)^{\alpha+1}}\right]^{\delta_{ij}}\left[\left(1+\frac{L_j^{vs_j^p}}{\beta}\right)^{-\alpha}\right]^{1-\delta_{ij}}\right\}.$$

$$=\prod_{j=1}^{k}\prod_{i=1}^{n_j}\left\{\left[\frac{\alpha vs_j^p\, t_{ij}^{vs_j^p-1}}{\beta+t_{ij}^{vs_j^p}}\right]\left[\frac{\beta}{\beta+t_{ij}^{vs_j^p}}\right]^{\alpha}\right\}^{\delta_{ij}}\left[\frac{\beta}{\beta+L_j^{vs_j^p}}\right]^{\alpha(1-\delta_{ij})} \tag{2.3}$$

where δ_{ij} is an indicator variable such that:

$$\delta_{ij}=\begin{cases}1 & for \quad t_{ij}\le L_j\\ 0 & for \quad t_{ij}>L_j\end{cases}\qquad \text{for all } i=1,\ldots\ldots,n_j,\ j=1,\ldots\ldots k. \tag{2.4}$$

It is well known that the ML estimator of v, p, β *and* α are obtained by maximizing the logarithm of likelihood function, which can be written in the form:

$$\ln L(v,p,\beta,\alpha|\underline{t}) = \ln\alpha\sum_j\sum_i \delta_{ij} + \ln v\sum_j\sum_i \delta_{ij} + p\sum_j\sum_i \delta_{ij}\ln s_j$$

$$+\sum_j\left(vs_j^p - 1\right)\sum_i \delta_{ij}\ln t_{ij} - \sum_j\sum_i \delta_{ij}\ln\left(\beta + t_{ij}^{vs_j^p}\right) + \alpha\ln\beta\sum_j\sum_i \delta_{ij}$$

$$-\alpha\sum_j\sum_i \delta_{ij}\ln\left(\beta + t_{ij}^{vs_j^p}\right) + \alpha\sum_j\sum_i\left(1-\delta_{ij}\right)\ln\beta$$

$$-\alpha\sum\sum\left(1-\delta_{ij}\right)\ln\left(\beta + L_j^{vs_j^p}\right) . \qquad (2.5)$$

The derivatives of the logarithm of likelihood function with respect to v, p, β and α respectively are given by:

$$\frac{\partial \ln L}{\partial v} = \frac{1}{v}\sum_j\sum_i \delta_{ij} + \sum_j s_j^p \sum_i \delta_{ij}\ln t_{ij} - \sum_j\sum_i \delta_{ij}\frac{s_j^p t_{ij}^{vs_j^p}\ \ln t_{ij}}{\left(\beta + t_{ij}^{vs_j^p}\right)}$$

$$-\alpha\sum_j\sum_i \delta_{ij}\frac{s_j^p t_{ij}^{vs_j^p}\ \ln t_{ij}}{\left(\beta + t_{ij}^{vs_j^p}\right)} - \alpha\sum_j\sum_i\left(1-\delta_{ij}\right)\frac{s_j^p L_j^{vs_j^p}\ \ln L_j}{\beta + L_j^{vs_j^p}} .$$

(2.6)

$$\frac{\partial \ln L}{\partial p}=\sum_j\sum_i\delta_{ij}\ln s_j+v\sum_j s_j^p \ln s_j\sum_i\delta_{ij}\ln t_{ij}$$

$$-(1+\alpha)\sum_j\sum_i\delta_{ij}\frac{vt_{ij}^{vs_j^p}\ln t_{ij}s_j^p\ln s_j}{\left(\beta+t_{ij}^{vs_j^p}\right)}$$

$$-\alpha\sum_j\sum_i\left(1-\delta i_{ij}\right)\frac{vL_j^{vs_j^p}\ln L_j s_j^p\ln s_j}{\left(\beta+L_j^{vs_j^p}\right)}\ . \tag{2.7}$$

$$\frac{\partial \ln L}{\partial \beta}=-\sum_j\sum_i\frac{\delta_{ij}}{\left(\beta+t_{ij}^{vs_j^p}\right)}+\frac{\alpha}{\beta}\sum_j\sum_i\delta_{ij}-\alpha\sum_j\sum_i\frac{\delta_{ij}}{\left(\beta+t_{ij}^{vs_j^p}\right)}$$

$$+\frac{\alpha}{\beta}\sum_j\sum_i\left(1-\delta_{ij}\right)-\alpha\sum_j\sum_i\frac{\left(1-\delta_{ij}\right)}{\left(\beta+L_j^{vs_j^p}\right)}\ . \tag{2.8}$$

$$\frac{\partial \ln L}{\partial \alpha}=\frac{1}{\alpha}\sum_j\sum_i\delta_{ij}+\ln\beta\sum_j\sum_i\delta_{ij}-\sum\sum\delta_{ij}\ln\left(\beta+t_{ij}^{vs_j^p}\right)$$

$$+\sum_j\sum_i\left(1-\delta_{ij}\right)\ln\beta-\sum_j\sum_i\left(1-\delta_{ij}\right)\ln\left(\beta+L_j^{vs_j^p}\right)\ . \tag{2.9}$$

Therefore the MLE may be found by setting (2.6), (2.7), (2.8) and (2.9) equal to zero. As shown they are nonlinear equations, their solutions are numerically obtained by using Newton-Raphson method as will be seen later. They are solved numerically to obtain v, p, β, α.

The asymptotic variance-covariance matrix of the estimators of v, p, β, α is obtained depending on the inverse fisher information matrix using the second derivatives of the logarithm of likelihood function where:

$$\frac{\partial^2 lnL}{\partial p^2} = v\sum_j (lns_j)^2 s_j^p \sum_i \delta_{ij} lnt_{ij}$$

$$-(1+\alpha)\sum_j\sum_i \delta_{ij}\left\{\frac{\left(\beta+t_{ij}^{vs_j^p}\right)\left(vlnt_{ij}lns_j\right)\left[s_j^p vs_j^p t_{ij}^{vs_j^p} lns_j lnt_{ij} + t_{ij}^{vs_j^p} s_j^p lns_j\right]}{\left(\beta+t_{ij}^{vs_j^p}\right)^2} - \frac{\left[vlnt_{ij}lns_j s_j^p t_{ij}^{vs_j^p}\right]^2}{\left(\beta+t_{ij}^{vs_j^p}\right)^2}\right.$$

$$\left.-\alpha\sum_j\sum_i\left\{\frac{\left(\beta+L_j^{vs_j^p}\right)\left(vlnL_j lns_j\right)\left(s_j^p vs_j^p L_j^{vs_j^p} lns_j lnL_j + L_j^{vs_j^p} s_j^p lns_j\right)}{\left(\beta+L_j^{vs_j^p}\right)^2} - \frac{\left[vlnL_j lns_j s_j^p L_j^{vs_j^p}\right]}{\left(\beta+L_j^{vs_j^p}\right)^2}\right\}\right(1-$$

(2.10)

$$\frac{\partial^2 \ln L}{\partial v^2} = -\frac{1}{v^2}\sum_j\sum_i \delta_{ij}$$

$$-(1+\alpha)\sum_j\sum_i \delta_{ij}\frac{\left(\beta + t_{ij}^{vs_j^p}\right)\left[\left(s_j^p\right)^2\left(\ln t_{ij}\right)^2 t_{ij}^{vs_j^p}\right] - \left[s_j^p t_{ij}^{vs_j^p}\ \ln t_{ij}\right]^2}{\left(\beta + t_{ij}^{vs_j^p}\right)^2}$$

$$-\alpha\sum_j\sum_i\left(1-\delta_{ij}\right)\frac{\left(\beta + L_j^{vs_j^p}\right)\left[\left(s_j^p\right)^2\left(\ln L_j\right)^2 L_j^{vs_j^p}\right] - \left[s_j^p L_j^{vs_j^p}\ \ln L_j\right]^2}{\left(\beta + L_j^{vs_j^p}\right)^2}.$$

(2.11)

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{1}{\alpha^2}\sum_j\sum_i \delta_{ij} \qquad (2.12)$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \sum_j\sum_i \frac{\delta_{ij}}{\left(\beta + t_{ij}^{vs_j^p}\right)^2} - \frac{\alpha}{\beta^2}\sum_j\sum_i \delta_{ij} + \alpha\sum_j\sum_i \frac{\delta_{ij}}{\left(\beta + t_{ij}^{vs_j^p}\right)^2}$$

$$-\frac{\alpha}{\beta^2}\sum_j\sum_i\left(1-\delta_{ij}\right) + \alpha\sum_j\sum_i\frac{\left(1-\delta_{ij}\right)}{\left(\beta + L_j^{vs_j^p}\right)^2} \qquad (2.13)$$

$$\frac{\partial^2 \ln L}{\partial\alpha\partial\beta}=\frac{\sum_j\sum_i\delta_{ij}}{\beta}-\sum_j\sum_i\frac{\delta_{ij}}{\left(\beta+t_{ij}^{vs_j^p}\right)}+\sum_j\sum_i\left(1-\delta_{ij}\right)\left[\frac{1}{\beta}-\frac{1}{\left(\beta+L_j^{vs_j^p}\right)}\right]. \tag{2.14}$$

$$\frac{\partial^2 \ln L}{\partial\alpha\partial v}=-\sum_j\sum_i\delta_{ij}\frac{s_j^p t_{ij}^{vs_j^p}\ \ln t_{ij}}{\left(\beta+t_{ij}^{vs_j^p}\right)}-\sum_j\sum_i\left(1-\delta_{ij}\right)\frac{s_j^p L_j^{vs_j^p}\ \ln L_j}{\left(\beta+L_{ij}^{vs_j^p}\right)}. \tag{2.15}$$

$$\frac{\partial^2 \ln L}{\partial v\partial p}=\sum_j s_j^p \ln s_j\sum_i\delta_{ij}\ln t_{ij}-(1+\alpha)\sum_j\sum_i\delta_{ij}$$

$$\left\{\frac{\left(\beta+t_{ij}^{vs_j^p}\right)\ln t_{ij}\left(s_j^p vs_j^p t_{ij}^{vs_j^p}\ \ln s_j \ln t_{ij}+t_{ij}^{vs_j^p}\ s_j^p \ln s_j\right)}{\left(\beta+t_{ij}^{vs_j^p}\right)^2}-\frac{v\ln s_j\left(s_j^p t_{ij}^{vs_j^p}\ \ln t_{ij}\right)^2}{\left(\beta+t_{ij}^{vs_j^p}\right)^2}\right\}$$

$$-\alpha\sum_j\sum_i\left\{\frac{\left(\beta+L_j^{vs_j^p}\right)\ln L_j\left(s_j^p vs_j^p L_j^{vs_j^p}\ \ln s_j \ln L_j+L_j^{vs_j^p}\ s_j^p \ln s_j\right)}{\left(\beta+L_j^{vs_j^p}\right)^2}\right.$$

$$\left.\frac{v\ln s_j\left(s_j^p L_j^{vs_j^p}\ln L_j\right)^2}{\left(\beta+L_j^{vs_j^p}\right)^2}\right\}(1-\delta_{ij}). \qquad (2.16)$$

$$\frac{\partial^2 \ln L}{\partial\alpha\partial p}=-\sum_j\sum_i\delta_{ij}\frac{v\,t_{ij}^{vs_j^p}\ln t_{ij}s_j^p\ln s_j}{\left(\beta+t_{ij}^{vs_j^p}\right)}-\sum\sum(1-\delta_{ij})\frac{vL_j^{vs_j^p}\ln L_j s_j^p\ln s_j}{\left(\beta+L_j^{vs_j^p}\right)}.$$

(2.17)

$$\frac{\partial^2 \ln L}{\partial\beta\partial p}=(1+\alpha)\sum_j\sum_i\delta_{ij}\frac{v\,t_{ij}^{vs_j^p}\ln t_{ij}s_j^p\ln s_j}{\left(\beta+t_{ij}^{vs_j^p}\right)^2}$$

$$+\alpha\sum_j\sum_i(1-\delta_{ij})\frac{v\,L_j^{vs_j^p}\ln L_j s_j^p\ln s_j}{\left(\beta+L_j^{vs_j^p}\right)^2}\ . \qquad (2.18)$$

$$\frac{\partial^2 \ln L}{\partial\beta\partial v}=(1+\alpha)\sum_j\sum_i\delta_{ij}\frac{s_j^p t_{ij}^{vs_j^p}\ln t_{ij}}{\left(\beta+t_{ij}^{vs_j^p}\right)^2}+\alpha\sum_j\sum_i(1-\delta_{ij})\frac{s_j^p L_j^{vs_j^p}\ln L_j}{\left(\beta+L_j^{vs_j^p}\right)^2}. \tag{2.19}$$

The asymptotic Fisher-Information matrix can be written as follows:

$$I=-\begin{bmatrix}\frac{\partial^2 \ln L}{\partial v^2} & \frac{\partial^2 \ln L}{\partial v\partial p} & \frac{\partial^2 \ln L}{\partial v\partial\alpha} & \frac{\partial^2 \ln L}{\partial v\partial\beta}\\ \frac{\partial^2 \ln L}{\partial p\partial v} & \frac{\partial^2 \ln L}{\partial p^2} & \frac{\partial^2 \ln L}{\partial p\partial\alpha} & \frac{\partial^2 \ln L}{\partial p\partial\beta}\\ \frac{\partial^2 \ln L}{\partial\alpha\partial v} & \frac{\partial^2 \ln L}{\partial\alpha\partial p} & \frac{\partial^2 \ln L}{\partial\alpha^2} & \frac{\partial^2 \ln L}{\partial\alpha\partial\beta}\\ \frac{\partial^2 \ln L}{\partial\beta\partial v} & \frac{\partial^2 \ln L}{\partial\beta\partial p} & \frac{\partial^2 \ln L}{\partial\beta\partial\alpha} & \frac{\partial^2 \ln L}{\partial\beta^2}\end{bmatrix}. \tag{2.20}$$

The MLE $\hat{v},\hat{p},\hat{\alpha}$ *and* $\hat{\beta}$ have an asymptotic variance-covariance matrix defined by inverting the above information matrix.

Practically, it is difficult to use results obtained at accelerated conditions to make prediction about the product performance over time at the use or design conditions. When making prediction from an ALT, one must make strong assumptions about the adequacy of the ALT process to describe the use process. Selection of the accelerated model is the most+ important difficulty. This model relates one or more parameter(s) to the stress levels that are to be applied to the testing items, it should be physically suitable for the item or product being tested and the type of stress being applied to accelerate failures (13).

The inverse power law model, which is the most commonly used in practice, is considered.

To predict the value of the scale parameter ϕ_u , under stress c_u , the invariance property of MLE is used. The MLE of the scale parameter of Generalized Burr distribution, $\hat{\phi}_u$ can be derived by using the following equation:

$$\hat{\phi}_u = \hat{\nu}\, s_u^{\hat{p}}, \tag{2.21}$$

where

$$s_u = c^* / c_u .$$

Furthermore, the MLE of the reliability function under usual conditions,

$$\hat{R}_u(t_0) = \left(1 + \frac{t_0^{\hat{\phi}}}{\hat{\beta}} \right)^{-\hat{\alpha}} . \tag{2.22}$$

In section 5, the scale parameter and the reliability function at different mission times are predicted under design stress $v_u = 0.5$.

3- The Confidence Limits of MLEs:

The maximum likelihood method provides a single point estimate for a population value. A confidence interval indicates the uncertainty in an estimate calculated from sample data, it encloses the population value with a specified high probability. Confidence intervals indicates how precise or imprecise estimates are where they reflect the random scatter in the data. The length of such an interval indicates if that corresponding estimates is accurate enough for practical purposes. Confidence intervals are generally wider than inexperienced data analysts expect; so confidence intervals help one avoid thinking that estimates are closer to the true value than they really are [14].

As indicated by Vander Wiel and Meeker (1990) [15], the most common method to set confidence bounds for the parameters is to use the large-sample (asymptotic) normal distribution of the ML estimators.

To define a confidence interval for a population value ω ; suppose $\omega_* = \omega_*(y_1, \ldots\ldots\ldots\ldots, y_n)$ and $\omega_{**} = \omega_{**}(y_1, \ldots\ldots\ldots\ldots, y_n)$ are functions of the sample data $y_1, \ldots\ldots\ldots\ldots, y_n$ such that:

$$p_\omega(\omega_* \le \omega \le \omega_{**}) = \gamma,$$

where the interval $[\omega_*, \omega_{**}]$ is called a two sided *$100\gamma\%$* confidence interval for ω , where ω_* and ω_{**} are the random lower and upper confidence limits that enclose ω with probability γ .

For large sample size, the maximum likelihood estimates under appropriate regularity conditions, are consistent and asymptotically normally distributed. Therefore, the two-sided approximate $100\gamma\%$ confidence limits for the maximum likelihood estimate $\hat{\omega}$ of a population value ω can be obtained by :

$$p\left[-z \le \frac{\hat{\omega}-\omega}{\sigma(\hat{\omega})} \le z\right] \cong \gamma, \qquad (3.1)$$

where z is the $\left[\frac{100(1-\gamma)}{2}\right]^{th}$ standard normal percentile. Therefore, the two-sided approximate $100\gamma\%$ confidence limits for ν, p, α, β will be respectively, as follows:

$$\left.\begin{array}{lll} L_\nu = \hat{\nu} - z\sigma(\hat{\nu}) & , & U_\nu = \hat{\nu} + z\sigma(\hat{\nu}) \\ L_p = \hat{p} - z\sigma(\hat{p}) & , & U_p = \hat{p} + z\sigma(\hat{p}) \\ L_\alpha = \hat{\alpha} - z\sigma(\hat{\alpha}) & , & U_\alpha = \hat{\alpha} + z\sigma(\hat{\alpha}) \\ L_\beta = \hat{\beta} - z\sigma(\hat{\beta}) & , & U_\beta = \hat{\beta} + z\sigma(\hat{\beta}) \end{array}\right] \quad (3.2)$$

4-Optimum Constant-stress Test Plans:

Most of the test plans are equally-spaced test stresses i.e. the same numbers of test units are allocated to each level of stress. Such type of test plans are usually inefficient for estimating the mean life at design stress (Yang,1994) [16].

The optimum test plan for products having a generalized Burr lifetime distribution is derived in which the choice of the allocation to each stress will be investigated such that the GAV of the MLE of the model parameters at use stress is minimized.

Generalized Asymptotic Variance of the Model Parameters: (an optimality criterion)

The GAV of the MLE of the model parameters is the reciprocal of the determinant of the Fisher information matrix denoted by I (Bai, *et al.*, 1993) [17]. That is:

$$GAV(\hat{\nu}, \hat{p}, \hat{\alpha}, \hat{\beta}) = |I|^{-1} \quad (4.1)$$

Thus, minimization of the GAV is equivalent to maximization of the determinant of I. The Newton-Raphson method is applied to determine numerically the best choice of the censoring time at each level of stress which minimizes the GAV as defined previously. Accordingly, the corresponding optimal censoring time at each level of stress can be obtained.

From equation (2.20)

$$I = \begin{bmatrix} -a_{11} & -a_{12} & -a_{13} & -a_{14} \\ -a_{12} & -a_{22} & -a_{23} & -a_{24} \\ -a_{13} & -a_{23} & -a_{33} & -a_{34} \\ -a_{14} & -a_{24} & -a_{34} & -a_{44} \end{bmatrix} \tag{4.2}$$

then

$$\begin{aligned} |I| = {} & \left(a_{11}a_{22}a_{33}a_{44} - a_{11}a_{22}a_{34}^2 - a_{11}a_{23}^2a_{44} + a_{11}a_{23}a_{34}a_{24} \right. \\ & \left. + a_{11}a_{24}a_{23}a_{34} - a_{11}a_{24}^2a_{33}\right) - \left(a_{12}^2a_{33}a_{44} - a_{12}^2a_{34}^2 \right. \\ & \left. - a_{12}a_{23}a_{13}a_{44} + a_{12}a_{23}a_{34}a_{14} + a_{12}a_{24}a_{13}a_{34} - a_{12}a_{24}a_{33}a_{14}\right) \\ & + \left(a_{13}a_{12}a_{23}a_{44} - a_{13}a_{12}a_{34}a_{24} - a_{13}^2a_{22}a_{44} + a_{13}a_{22}a_{34}a_{14} \right. \\ & \left. + a_{13}^2a_{24}^2 - a_{13}a_{24}a_{23}a_{14}\right) - \left(a_{14}a_{12}a_{23}a_{34} - a_{14}a_{12}a_{33}a_{24} \right. \\ & \left. - a_{14}a_{22}a_{13}a_{34} + a_{14}a_{22}a_{33}a_{14} + a_{14}a_{23}a_{13}a_{24} - a_{14}^2a_{23}^2\right). \end{aligned} \tag{4.3}$$

So, by setting the following equations equal to zero, L_j, $j = 1,2,3$ can be optimally determined by solving them simultaneously and applying the Newton-Raphson method:

$$\frac{\partial |I|}{\partial L_j}, j = 1,2,3. \tag{4.4}$$

5-Numerical Results of Simulation Studies:

The main aim of this section is to make a numerical investigation to illustrate the theoretical results of both estimation and optimal design problems. Several data sets are generated from Generalized Burr distribution for a combination of the true parameter values of v, p, α and β and for sample sizes 100,200,300,400 and 500 using 500 replications for each sample size. It is assumed that k=3 i.e. there are only three different levels of stress $c_1 = 1, c_2 = 1.5, c_3 = 2$,which are higher than the stress at use condition; $c_u = 0.5$ Numbers of test units are allocated to each level of stress $(n_j, j = 1,2,3)$ follow the sub sample-proportions $\pi_j, j = 1,2,3$, where $\pi_1 = 0.5$, $\pi_2 = 0.3$, $\pi_3 = (1-(\pi_1 + \pi_2))$, the pre-specified censoring times are $L_1 = 185, L_2 = 2000$ and $L_3 = 12000$; (Type-I censoring).

The true parameter values of v, p, α, β used in this simulation study are (0.6,0.9,1.6,30) to generate $(t_{ij}, i = 1,........, n_j, j = 1,2,3)$. Computer programs are derived depending on Mathematica 5.0 using the iterative technique of Newton-Raphson method to solve the derived nonlinear logarithmic likelihood equations in (2.5),(2.6),(2.7) and (2.8) simultaneously.

Once the values of v, p, β and α are obtained, these estimators are used to obtain; depending on equation (2.21) and letting the design stress, $c_u = 0.5$, the scale parameter under this stress, ϕ_u, is predicted as $\hat{\phi}_u = \hat{v}\, s_u^{\hat{p}}$ where $s_u = c^* / c_u$. Also, the reliability function is predicted for different values of mission times under use condition using (2.22).

Evaluating the performance of the estimators of v, p, α, β has been considered through some measurements of accuracy. In order to study the precision and variation of maximum likelihood estimators, it is convenient to use, firstly, the mean relative absolute bias (MRA Bias); which is the mean of absolute difference between the estimated parameter and its true value divided by its true value. The second one is the relative absolute bias (RA Bias); which is the absolute difference between the estimated parameter and its true value divided by its true value. The third one is the mean square error (MSE); which is the mean of the square difference between the estimated parameter and its true value. Also the relative error (RE of the estimator); which is the square root of the MSE of the estimator divided by its true value.

Table (1) demonstrates the average number of units failed at each level of stress; $\bar{r}_1, \bar{r}_2$ and $\bar{r}_3$ .Also Table(1) summarizes the results of solving the ML equations of v, p, β α in type I censoring for different sample sizes with their MRA Bias, RA Bias, MSE and RE. The numerical results indicate that the ML approximate the true values of the parameters as the sample size increases. Also, as shown in the numerical results the MRA Bias, the RA Bias the MSE and the RE are decreasing when the sample size is increasing.

Table (2) shows the asymptotic variance-covariance matrix for the same different sample sizes. As shown in the table, the asymptotic variances of the estimators are decreasing as n is getting to be large.

Table (3) presents the predicted values of the scale parameter and the reliability function. In general it is known that the reliability decreases when the mission time (t_0) increases. The results show that reliability reduces when the mission time increases from 3.6 to 4 . Therefore, the results get better in the sense that the aim of an ALT experiments is to get .large number of failures (reduce the reliability) of the device of high reliability. Also the same table shows that the relative absolute bias RA Bias (the absolute difference between the predicted reliability function and its true value divided by its true value) is reducing when the sample size is getting to be large.

To obtain the confidence intervals for the four parameters v, p, β and α , the equations (3.2) are used for each parameter, five different sized samples of n=100(100)500 are considered with parameters v=0.6, p=0.9, α=1.6 and β=30. Table (4) demonstrates the two-sided confidence limits with confidence level 95% of the parameters. As shown from the results, the interval of the estimator is getting to be narrow as the sample size increases.

Optimum test plans are developed numerically, it can be observed from the numerical results presented in Table (5), that the optimum test plans do not specify the same censoring time to each stress. Also table (5) includes the optimal censoring time of each level of stress for the considered different sized samples represented by L_1^*, L_2^* *and* L_3^* which minimize the GAV of the MLE of the model parameters. As indicated from the results, the optimal GAV of the MLE of the model parameters is decreased as the sample size n is increasing. Also, the corresponding optimal average number of units failed at each level of stress; $\bar{r}_1^*, \bar{r}_2^*$ and $\bar{r}_3^*$, respectively, are presented in this table.

Table(1): The Estimates, MRA Bias, RA Bias, MSE, RE of the Parameters $v = 0.6, p = 0.9, \alpha = 1.6, \beta = 30$ for Different Sample Size

n	$\bar{r}_1$	$\bar{r}_2$	$\bar{r}_3$	Estimates	MRA Bias	RAB	MSE	RE
100	40	24	15	$\hat{v}$=0.61061 $\hat{p}$=0.87909 $\hat{\alpha}$=3.28447 $\hat{\beta}$=60.3374	0.115427 0.14785 1.24341 1.1344	0.0176675 0.0232342 1.0528 1.01125	0.0090156 0.0277703 22.4422 4566..47	0.155504 0.189565 1.44234 1.11996
200	80	49	31	$\hat{v}$=0.60559 $\hat{p}$=0.88857 $\hat{\alpha}$=2.26362 $\hat{\beta}$=42.863	0.084315 0.096839 0.586244 0.557829	0.0093125 0.0126951 0.414761 0.428765	0.0045704 0.0121487 2.07248 589.843	0.111634 0.124043 0.635978 0.566612
300	121	73	46	$\hat{v}$=0.606024 $\hat{p}$=0.89489 $\hat{\alpha}$=2.04081 $\hat{\beta}$=38.6156	0.0759871 0.0846725 0.46873 0.438146	0.0100401 0.0056757 0.275506 0.287185	0.0034325 0.0093151 1.2174 362.247	0.0966753 0. 107851 0.540648 0.492879
400	160	97	62	$\hat{v}$=0.603205 $\hat{p}$=0.89345 $\hat{\alpha}$=2.01159 $\hat{\beta}$=37.4267	0.0682417 0.0735566 0.433795 0.387257	0.0053409 0.0072751 0.257245 0.247558	0.0028660 0.0077635 1.2026 360.874	0.0887509 0.0986184 0.545156 0.507570
500	200	122	77	$\hat{v}$=0.601554 $\hat{p}$=0.88825 $\hat{\alpha}$=1.90164 $\hat{\beta}$=35.4214	0.0610528 0.0663584 0.355289 0.319755	0.0025901 0.0130584 0.188529 0.180714	0.0021315 0.0057230 0.667454 187.195	0.076749 0.0851687 0.429617 0.386262

Table(2): Asymptotic Variances and Covariances of Estimates for Different Samples Size of the Parameters $\nu = 0.6, p = 0.9, \alpha = 1.6, \beta = 30$ Using Type-I Censoring

n	Variance-Covarianc Matrix			
	ν	p	α	β
100	0.0012647	-0.0000418	-0.0018824	0.139304
	-0.0000418	0.0233747	-0.0007417	-0.018308
	-0.0018824	-0.0007417	0.0975838	2.26091
	0.139304	-0.018308	2.26091	107.422
200	0.000916	-0.0000407	-0.0021869	0.0847654
	-0.0000407	0.0123173	-0.0001817	-0.0092713
	-0.0021869	-0.0001817	0.085435	1.86178
	0.0847654	-0.0092713	1.86178	73.6862
300	0.00068928	-0.00002084	-0.00194728	0.0510573
	-0.00002084	0.00843148	-0.00024963	-0.0085341
	-0.00194728	-0.00024963	0.0654723	1.40466
	0.0510573	-0.0085341	1.40466	51.5389
400	0.00056689	-0.00001166	-0.00195202	0.0339775
	-0.00001166	0.00636904	-0.00020653	-0.00612068
	-0.00195202	-0.00020653	0.0596891	1.24747
	0.0339775	-0.00612068	1.24747	42.9935
500	0.00050886	-0.00000724	-0.00211494	0.0212025
	-0.00000724	0.00533451	-0.00019042	-0.00541578
	-0.00211494	-0.00019042	0.0586785	1.17434
	0.0212025	-0.00541578	1.17434	37.2923

Table(3): The Estimated Scale Parameter and Reliability Under Use Condition at Different Samples Size When
$v = 0.6, p = 0.9, \alpha = 1.6$ *and* $\beta = 30$

n	$\hat{\phi}_u$	t_0	$\hat{R}_u(t_0)$	Relative Bias
100	1.41174	3.6	0.728812	0.0188026
		3.8	0.711647	0.0211551
		4	0.694646	0.023662
200	1.41287	3.6	0.739624	0.00424657
		3.8	0.723308	0.005115
		4	0.707165	0.00606656
300	1.42243	3.6	0.738474	0.00579526
		3.8	0.722117	0.00675373
		4	0.705944	0.00778216
400	1.41387	3.6	0.737479	0.00713438
		3.8	0.727027	0.00801894
		4	0.705106	0.0089599
500	1.40302	3.6	0.741508	0.00170966
		3.8	0.725602	0.00196072
		4	0.709885	0.00224339

Table(4):Confidence Bounds of the Estimates at Confidence Level 95% When $v = 0.6, p = 0.9, \alpha = 1.6$ and $\beta = 30$

n	Parameter	Estimates	Standard Devision	Lower Bound	Upper Bound
100	v	0.610601	0.0355623	0.540898	0.680303
	p	0.879089	0.152888	0.579429	1.17875
	α	3.28447	0.312384	2.6722	3.89675
	β	60.3374	10.3644	40.0231	80.6517
200	v	0.605587	0.030266	0.546266	0.664909
	p	0.888574	0.110983	0.671047	1.1061
	α	2.26362	0.292293	1.69072	2.83651
	β	42.863	8.58407	26.0382	59.6877
300	v	0.606024	0.0262541	0.554566	0.657482
	p	0.894892	0.0918231	0.714919	1.07487
	α	2.04081	0.255876	1.53929	2.54233
	β	38.6156	7.17906	24.5446	52.6865
400	v	0.603205	0.0238095	0.556538	0.649871
	p	0.893452	0.0798062	0.737032	1.04987
	α	2.01159	0.244313	1.53274	2.49045
	β	37.4267	6.55695	24.5751	50.2784
500	v	0.601554	0.0225579	0.557341	0.645767
	p	0.888247	0.0730377	0.745093	1.0314
	α	1.90164	0.242237	1.42686	2.37643
	β	35.4214	6.10674	23.4522	47.3906

Table(5):The Results of Optimal Design of the Life Test for Different Sized Samples Under Type-I Censoring in Constant-Stress FALT Given L_1=185, L_2=2000 and L_3=12000

n	$\bar{r}_1$	$\bar{r}_2$	$\bar{r}_3$	L_1^*	L_2^*	L_3^*	$\bar{r}_1^*$	$\bar{r}_2^*$	$\bar{r}_3^*$	GAV
100	40	24	15	252.41	886.43	185395	42	21	19	0.000076
200	80	49	31	141.30	1007.58	34432.3	76	44	34	0.000012
300	121	73	46	128.07	841.02	26183.3	111	64	50	0.000003
400	160	97	62	122.35	841.25	20139.9	146	85	65	0.000001
500	200	122	77	115.71	820.84	16845.8	180	106	80	0.0000006

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استخدام اختبارات الحياة المعجلة لتقدير معالم توزيع

بيير العام فى حالة العينات المراقبة من النوع الأول

جميلـــة نصـــر

هذا البحث يقوم بمعالجة اختبارات الحياة المعجلة الثابتة التامة فى حالة ثلاثة مستويات من الضغوط فى حالة البيانات المبتورة . تم تقسيم الوحدات محل الاختبار ، بحيث تعمل الوحدة محل الاختبار فى كل مجموعة تحت ظرف واحد ثابت طول فترة الاختبار . وقد تم استخدام طريقة الإمكان الأعظم كأسلوب لتقدير معالم التوزيع ، فى حالة العينات المراقبة من النوع الأول (Type-II cen- coring) ، وكذلك الحصول على مصفوفة التباين والتغاير وذلك لمقدرات معالم التوزيع . وقد تم حساب تقديرات للحدود الدنيا والعليا "فترات الثقة" المناظرة لكل معلمة ، وكذلك تم التوصل للأوقات المثلى التى يتم التوقف عندها إجراء كل من التجارب فى المجموعات الثلاث التى تم تقسيم العينة الكلية لها . ثم تم استخدام أسلوب المحاكاة فى توضيح النتائج من خلال مثال عددى .